McEvoy on Benacerraf's problem and the epistemic role puzzle

Jody Azzouni
Tufts University
jodyazzouni@mindspring.com
Forthcoming in *New Perspectives on the Philosophy of Paul Benacerraf: Truth, Objects, Infinity* (ed. Fabrice Pataut). Springer: *Logic, Epistemology And the Unity of Science*, 2015.

1. *Benacerraf's problem*.

Benacerraf's problem is justly famous. It's had a major influence on the philosophy of mathematics right from its initial appearance,[1] an influence that continues up through the present moment. In its author's supernaturally elegant prose, it lays out a tension between the possibility of an epistemic access to abstracta and the apparent semantics (truth conditions) of mathematical statements about those entities. Given a causal construal of epistemic access, on the one hand, it seems that we can't have any epistemic access to the objects that our true mathematical statements must be about because those objects are causally inefficacious and causally insensitive; on the other hand, the mathematical truths in question are genuinely *about* those objects, and somehow we are adept at identifying some of the true mathematical statements and some of the false ones.

Benacerraf's problem long outlasted the faddish "causal theory of knowledge" that he originally couched it in terms of. Field (1989, 26), among others, generalized Benacerraf's problem by writing:

> Benacerraf's challenge … is to provide an account of the mechanisms that explains how our beliefs about these remote entities can so well reflect the facts about them. The idea is that if it appears in principle impossible to explain this, then that tends to undermine the

[1] Benacerraf 1973.

belief in mathematical entities, despite whatever reason we might have for believing in them.

The challenge is now put in terms of mechanisms *of any sort*—they needn't be causal ones. What's important is that such mechanism are required to explain—at least in principle—how our beliefs about these "remote" entities so well reflect the facts about them. So what's being indicated is the worry that because the entities in question are typically abstract (e.g., they're not in space and time, they're not causally efficacious or causally sensitive), no possible mechanism can—*even in principle*—foot the bill.

And indeed, one strand of the widespread philosophical response to Benacerraf's problem is precisely to supply an "in principle" mechanism for these remote entities by denying their purported *remoteness*. Maddy (1990) suggests that at least some sets are *in* space and time, the set of the shoes that I'm wearing, for example, is claimed by her to be located where my shoes are at; she further suggests that I *see* this set each time I happen to gaze down at the shoes themselves. Field (similarly) suggests that points and spacetime regions aren't disagreeably remote because (after all) they're *in* spacetime; so too there's an *in principle* perceptual mechanism by which our beliefs about points and spacetime regions can so well reflect the facts about them.

Leaving aside the discussions of "in principle" mechanisms that (some) philosophers are so addicted to, Benacerraf's problem is recognizably a *modern* one, not visible to earlier philosophers concerned with mathematical entities. Plato, for example, can be seen as offering an *experiential*, if not perceptual, theory of the mechanisms involved. As disembodied souls we previously experienced the mathematical objects that some of us (in our current lives) nostalgically recollect. (*Those people* are called “mathematicians.”) A later view, roughly attributable to Descartes, is that mathematical ideas are innately imprinted in our minds. The worry about such imprints being reliable remains—it’s not solved by merely postulating that mathematical ideas are innate; it’s solved (by Descartes) using the hypothesis that an “honest-broker” deity is the (ultimate) source of these ideas. It’s that the metaphysics behind these earlier views are no longer respectable—so too, that certain epistemic accompaniments (e.g., “rational intuition”) aren’t respectable either—that’s allowed Benacerraf’s problem to emerge as a *problem* for contemporary philosophers of mathematics.

Meanwhile (elsewhere, in the sciences) a progressive understanding of the *actual* mechanisms of perception has emerged, especially in this and the last century. That is to say, the *actual mechanisms* behind our abilities to perceive those parts of the world that we can perceive are being uncovered. And this extends to our understanding of the instrumental mechanisms by which we’ve extended our sensory capacities; sophisticated instruments that gain us epistemic access to otherwise sensorily-remote parts of the universe (the very far away, the very small, for example) are themselves the subjects of intricate scientific studies.[2] Epistemic access (in empirical realms, anyway) is itself the subject of sophisticated science.

The remarkable fact about mathematics is that there is *nothing* corresponding to the scientific study of the *epistemic access* to its entities. There is, of course, serious cognitive-science studies of the (largely subpersonal) mental processes that occur when animals, children, and adults engage in mathematical ratiocination (e.g., when they count some things that are near

[2] See, e.g., Strobel and Heineman 1989, Barrett and Myers 2004.

them)[3]; but in lieu of Cartesian deities, these studies of arithmetical competence hardly count as studies of *epistemic access*.

An important corollary of the emergence of studies of epistemic access is the systematic recognition of *epistemic artifacts*. Epistemic artifacts are the ways that our means of access to objects distort our impressions of the properties of those objects. In vision science, this takes the form of a systematic study of "optical illusions," in particular, the study of what it is about our visual capacities that allows such illusions to occur.[4] A corresponding study of our instrumental access to objects occurs in the sciences; we learn how the mechanisms by which an instrument allows us to learn about its target objects are limited in various ways, how they generate false impressions of the properties of the objects, and so on.[5] Understood broadly: these are studies of how we make *mistakes* about the objects we're epistemically accessing in particular ways; and it's our (scientific) *understanding* of the epistemic mechanisms we use to gain access to those objects that explains how those mistakes arise.

In the mathematical sciences, by contrast, mistakes are invariably "proof-theoretic ones." I understand "proof-theoretic" broadly: it can be a matter of our failure to execute a computation correctly, by actually writing down the wrong numerals,[6] for example; but it can also be a matter of conceptualizing a class of objects the wrong way. What it never involves, however, is that the mechanism of our epistemic access to the *abstracta* under study is misleading, that our means of epistemic access *to those objects* is itself creating problems. There is no study of the nature of such mechanisms; there is no science of such; there are no cases where we say, for example: *Rational intuition* often fails in such and such circumstances because ….

[3] See, e.g., Carey 2009, especially chapter 4, for discussion of the literature.
[4] See, e.g., the essays in Rock 1997.
[5] See, e.g., Hacking 1983, chapter 11.
[6] See Azzouni 1994, Part I, §5 and §6 for details about these kinds of errors.

Notice that these disanalogies with respect to epistemic access in these respective areas, mathematics and the various empirical realms, don't even remotely involve skeptical considerations—as philosophers generally construe those considerations, anyway. The point is an "internal" one about how the sciences, and the corresponding sciences of their various kinds of epistemic access to the world, have developed. I'll return to the purported relationship of these considerations about scientific studies of epistemic access to philosophical skepticism later in the paper.

2. *The epistemic role puzzle*.

In my *Metaphysical Myths, Mathematical Practice* (1994), I argued that the influence of Benacerraf's problem on the philosophy of mathematics literature was deeply *nefarious*: Focus on *it* has obscured our view of the real philosophical issues and puzzles posed by standard mathematical practice. Attempting to establish this, however, involved emulating Benacerraf's achievement by posing a *different* philosophical puzzle, one that can be seen as derived (as it were) from Benacerraf's problem by *flipping* the angle of concern. Instead of worrying about how it is we're supposedly getting access to these "remote" objects, notice instead, as the concluding paragraphs of the last section indicated, that mathematical objects play no epistemic role whatsoever in mathematical practice. What does a philosophical focus on *that* lead to?

My various expositions of the epistemic role puzzle were invariably accompanied by a *joke*. I've written, more than once: (Imagine that mathematical objects ceased to exist sometime in 1968. Mathematical work went on as usual. Why wouldn't it?)[7] But one—I've since learned—should *never ever* make jokes in philosophical work. It's not merely that a dismayingly large

[7] E.g., Azzouni 1994, 56, Azzouni 2004a.

number of professionals become totally lost when that happens; it's also that what's been written can (*will*) be taken literally. Perhaps it didn't help that some years later Balaguer (1998, 132) wrote, echoing my earlier remarks, that: "If all the objects in the mathematical realm suddenly *disappeared*, nothing would change in the physical world."

Regardless. An interpretation of the epistemic role puzzle appeared in the literature that characterized it as a *modal* argument. The epistemic role puzzle was interpreted as trading on the construction of a possible world where there are no abstracta but the (empirical) world is otherwise exactly the same. This *makes no difference argument* has enjoyed a bit of discussion;[8] but the considerations both for and against "makes no difference" arguments are remote from what motivates the epistemic role puzzle.

First of all, I was offering a colorful thought experiment that was meant to draw the reader's attention to the considerations raised at the end of the last section: that epistemic access to the mathematical objects themselves (purportedly referred to by mathematical terms) plays no epistemic role in the practice of mathematics, neither (specifically) in how mathematical results are established, nor in (specifically) our studies of how mathematical practice operates. I wasn't suggesting the existence of a possible world in which there are no mathematical objects but mathematical practice goes on as before.[9]

---

[8] See Baker 2003 for the original coinage of the term "makes no difference," and for an attack on the argument. See Raley 2008 for a careful elucidation of the complex strands involved in these "MND" arguments. She there labels an argument against the existence of mathematical abstracta that turns on the epistemic role puzzle the "epistemic version of MND"—I won't be using this terminology. I should add that I'm unsympathetic to the various MND arguments, largely for the reasons Raley gives for rejecting them. I won't discuss these details further now.

[9] In Azzouni 1994, when I first offered this thought experiment I was neutral on the question of the existence of abstracta—mainly because I already saw no reason to accept Quine's criterion. I first officially publicized my nominalism, however, in Azzouni 2004b. I thought the challenge posed by the epistemic role puzzle remained the same despite this change in viewpoint precisely because it wasn't (in my view) a philosophical claim but instead an (overlooked) insight about

Baker (2010, 224) writes of my thought experiment that

> provoking the intuition that the existence of mathematical objects makes no difference by depending on thought-experiments whose conditions are conceptually impossible gives the Platonist plenty of ammunition for resistance.

Even if my aim were to ask the reader to imagine a possible world in which there are no mathematical objects, I hardly think that demands the reader to entertain conditions that are *conceptually impossible*. Perhaps this melodramatic phrasing is meant to allude to the common view that mathematical objects are metaphysically necessary. Even so, to imagine abstracta as not existing isn't to imagine something that's conceptually impossible. (Metaphysical necessity doesn't imply conceptual impossibility; surely we all know that by now.) Perhaps, instead, Baker means to suggest that imagining mathematical objects not to exist is to imagine certain mathematical truths (that are conceptually necessary) to instead be falsehoods. But this doesn't follow either. Our *understanding* of mathematical statements, and our understanding that such are true *doesn't* require that anything *exist*.[10] At least, this needs to be established by argument and not just assumed, as it seems to be by Baker.

3. *McEvoy on the epistemic role puzzle: His general strategy*.

Claiming that sets, points and regions of spacetime are located within our perceptual ranges, I've noted, responds to Benacerraf's problem. That it doesn't respond to the epistemic role puzzle shows the puzzles to be distinct—according to whatever vague principles of

---

ordinary mathematical practice.

[10] See Azzouni 2004b.

individuation that *philosophical problems* seem to obey, anyway. One can't merely claim that points, regions of space, or sets *can* be perceived as Maddy and Field do in order to meet the epistemic role puzzle. One has to show that perception of these things actually *does* play a role in mathematical practice—in the proving of theorems, for example. (And, as we all also know by now: contexts of discovery don't count.) So "in principle" epistemic access to otherwise artificially-designed entities isn't *relevant*.

McEvoy, however, isn't out to show that the epistemic role puzzles reduces to Benacerraf's problem *tout court*; he only wants to show that the epistemic role puzzle raises no new considerations that *Platonists* need worry about.[11]

McEvoy's argument for this claim is ingenious, intricate and detailed. Here's (roughly) how it goes. Consider the epistemic role puzzle all on its own. *Nothing* follows about the existence or nonexistence of mathematical abstracta. After all, the epistemic role puzzle is only an *epistemic* observation, that there's no *epistemic* role for mathematical abstracta. So it's clear that claim alone can't imply that there are no mathematical abstracta, and *this* means that other premises are needed. McEvoy, by surveying the options that seem compatible with what I've written on this, argues that any other candidate premises—that would do the job to enable the epistemic role puzzle to refute the existence of mathematical entities—would manage that job *all on their own*. Therefore, argumentatively speaking, the epistemic role puzzle is an idle wheel.[12] McEvoy (2012, 10) writes:

---

[11] McEvoy 2012, 4, footnote 4.

[12] This is almost his argument. Here's another rider he sometimes employs: where the epistemic role puzzle actually seems to do some work (in some versions of the argument against mathematical entities that McEvoy tries to mount on my behalf) it turns out that what it's doing is indistinguishable from the work Benacerraf's problem does.

The added premise, if ERP is to count as both a legitimate and novel challenge to platonism, must not be one the truth of which would itself refute platonism.

4. *Correspondence Truth and Quine's criterion, according to McEvoy*.

Unsurprisingly, the candidate additional premises that McEvoy considers as possible supplementations for the epistemic role problem involve metaphysical assumptions. Here's one:

> (AZ) The only non-epistemic role that abstracta can play is that of providing truth conditions for mathematical statements within a correspondence theory of truth, and the correspondence theory of truth is false.[13]

McEvoy (2012, 8) writes:

> If AZ is added, it is no longer ERP that creates the problem for the platonist: it's the fact that platonism assumes the correspondence theory of truth, which, given AZ, is false. … With the addition of AZ, though the argument from ERP does become valid, since AZ itself refutes platonism, this validity is purchased only at the cost of making ERP redundant.

So too, consider a different premise:

[13] McEvoy 2012, 7.

(AZ*) The only non-epistemic role that abstracta can play is that of entities over which the quantifiers of a suitably regimented best theory range, and this incurs a commitment to the untenable Quinean criterion of ontological commitment.[14]

McEvoy (2012, 9) similarly observes:

> Once again the problem is that if AZ* is added, it is no longer ERP that creates the problem for the platonist: it is the assumption that the platonist must assume the (*ex hypothesi*) false Quinean criterion. Once again, when we add the premise to render the argument valid, ERP does none of the heavy lifting.

What's an Azzouni to do? Well, here's what I suggest *this Azzouni* do: Deny that AZ and AZ* are to be interpreted as McEvoy does; in particular, McEvoy glosses the implications of rejecting the correspondence theory of truth and of rejecting Quine's criterion far too strongly. Appropriately glossed, *rejections* of these fundamental principles aren't sufficient on their own to refute Platonism. What's needed to refute Platonism is, along with the rejections of these principles, the epistemic role puzzle. I turn now to spelling out what I have in mind.

5. *What does rejecting correspondence truth and Quine's criterion* actually imply?

Let's start with the rejection of the correspondence theory of truth (encapsulated in AZ) that, according to McEvoy, *all by itself* can be used to refute Platonism. It should be admitted that the rejection of the correspondence theory of truth is *often* taken to be, as McEvoy seems to,

[14] McEvoy 2012, 8.

the rejection of it with respect to *every* sentence of our language. When a philosopher is moodily fanatical this way, competing candidate theories of truth are also characterized globally. Coherence theories of truth, so interpreted—for example—deny that *any* sentence corresponds to anything. Instead, *every* sentence (that's true) coheres—whatever that means exactly—with the other true sentences. So too, deflationary theories are interpreted by global truth fanatics as denying of *every* sentence that in describing it as true one is ascribing a "substantial" truth-property to it. Deflationism, characterized this way, is militantly anti-metaphysical in intent.[15]

These alternatives to correspondence truth fit McEvoy's bill perfectly: if a Platonist adopts one of *these* theories of truth, quick work can be made of a purported correspondence between mathematical truths and how it is with mathematical abstracta. But, of course, more *liberal* theories of truth are out there and flourishing. A candidate that's emerged recently is one or another species of pluralist truth[16]: Our language divides neatly into various discourses. Some operate according to correspondence principles of one or another sort, some operate according to coherence principles of one or another sort. Truth pluralists don't claim that Platonism is ruled out just because correspondence truth has been ruled out for the language *as a whole*. Specific arguments about mathematical discourse are called for to adjudicate this question.

I'm not a fan of pluralism; but I *am* a proponent of a liberal version of deflationism.[17] *Liberal deflationism* treats the truth predicate as a logical device of blind truth ascription that's neutral with respect to whether true statements correspond to anything or not. "Mickey Mouse is depicted as smarter than Sherlock Holmes," and "Barack Obama is more famous than Saul

[15] See, e.g., Horwich 1998.
[16] See Lynch 2004.
[17] See Azzouni 2006, Azzouni forthcoming.

Kripke," are both true; the second statement (in my view) corresponds to the way it is with several items in the world; this isn't true of the first statement.

This view of truth is neutral—as far as it goes—with respect to whether statements about abstracta are like statements about Mickey Mouse or like statements about Barack Obama (true because of correspondence facts or true for some other reason). Liberal deflationism is a candidate theory of truth that's opposed to global correspondence; a proponent of it will accept AZ, but AZ doesn't, therefore, refute Platonism. Additional premises are needed to manage this.

The same point can be made about Quine's criterion and McEvoy's AZ*, so I'll be brief. *Rejecting* Quine's criterion doesn't imply that quantifier statements[18] are *never* ontologically committing; it only implies that we can't assume that any particular quantifier statement *is* ontologically committing. Further premises, therefore, are needed as well to establish (or deny) that quantifier statements about abstracta are ontologically committing.

6. *The epistemic role puzzle to the rescue*.

So what do the additional premises, needed by the opponent of Platonism, look like? I imagine, of course, that there is more than one way to go here, but I want to sketch an argument that utilizes the epistemic role puzzle as the needed additional premise.

Start with the following *criterion for what exists*: Anything that exists is mind- and language-independent. Next step: How do *we* recognize that an object is mind- and language-independent? Answer: It has an *epistemic role*. Contrariwise, if an object has no epistemic role, then it's mind- and language-dependent, and therefore (by our criterion) it doesn't exist. Mathematical abstracta have no epistemic role. Conclusion: there are no mathematical abstracta.

[18] Statements of the form $(\exists x) \ldots x \ldots$.

I've defended detailed versions of this argument elsewhere,[19] and haven't the space to do it again here; I'll offer a couple of elucidations. First off, the criterion for what exists is one that I take us to have collectively adopted. It's not revealed by a conceptual analysis of ordinary words like "exist" or "there is" because the perceived meanings of those words are compatible with any number of criteria.[20]

Second, the argument as I've put it, may seem to draw too strong a conclusion. Perhaps all that's licensed is the weaker conclusion that "we have *no reason to believe* in abstract objects, not that there are no such objects."[21] I think the stronger conclusion is licensed, however, because it's reasonable for us to say: *there are no Ss* if we have no reason to believe in Ss. Again, this is perhaps not the place to fully argue for this conclusion, but let me say this in my defense: It's sometimes reasonable for us to assert: *There are no Ss*. I think, for example, that it is reasonable for me to say: There is no Santa Claus, there are no hobbits, there is no Harry Potter. I think it's *understating* what I'm licensed to say if I say only: I've no reason to believe in these things. The latter is true, of course, and *it* is what licenses me to say that none of these things exist. Having said this, I should modestly add: I *could* be wrong. There might *be* a Santa Claus, a Harry Potter, some hobbits here and there. But surely the fact that I've no reason to believe in these things is compatible both with my being able to draw the conclusion: there are none of these things *and* I might be wrong about this. Surely the epistemic situation is the same with respect to *abstracta*.

7. *Skepticism again*.

[19] E.g., in Azzouni 2004b, and in the appendix of the General Introduction in Azzouni 2010a.
[20] See Azzouni 2007, 2010b.
[21] McEvoy 2012, 3.

This last point somewhat naturally brings us back to something I raised at the beginning of this paper, that the considerations behind the epistemic role puzzle (and behind Benaceraff's problem) aren't skeptical ones. McEvoy denies this, and as he notes, he has good company: Gödel, Katz, Burgess and Rosen (among others, no doubt). Giving his reasons for why the epistemic role puzzle as a challenge to the existence of Platonic objects *is* analogous to skepticism, he writes (and I quote him nearly in full):

> The skeptical argument at play in this argument is that, for all we know, it is possible that there is, right now, no physical world, but that we are deluded into thinking there is one. This is a possibility since all of the evidence available in the event of the existence of the external world would be present in the skeptical world. The mathematician is similarly situated with respect to mathematical objects; given the lack of an epistemic role for mathematical objects, any evidence that would lead a mathematician to believe that there are mathematical entities would be present whether or not such entities exist. From this perspective, Azzouni's thought experiment begs the question against the argument from skeptical analogy: it assumes that the processes that produce our beliefs about the external world are currently operating reliably (i.e., they give us genuine knowledge of the external world). … But the parallel between external-world skepticism and mathematical skepticism that is relevant to the argument from skeptical analogy actually blocks this assumption. This parallel has to do with the possibility that we may *right now* be deceived about what is causing those experiences on the basis of which we infer the existence of physical reality, on the one hand, and mathematical reality on the other. Given this parallel, we cannot, as Azzouni does, hold constant our epistemic story of how

> these appearances are caused, and then ask what would happen if everything disappeared. This move is at once ruled out by the parallel that operates in the argument from skeptical analogy. We of course believe that we have knowledge of the external world, due to the reliable operation of our senses. (Similarly platonists believe that we have knowledge of the mathematical realm due to the reliable operation of intuition, or reason.) However, the point is that the falsity of this belief is compatible with our having a phenomenologically indistinguishable experience. This possibility of our having phenomenologically indistinguishable experiences regardless of whether entities of a certain kind exist is, after all, the point of skepticism; and the point of the argument from skeptical analogy is that this possibility obtains equally in the cases of mathematical and external-world knowledge.

How analogies between things may be drawn is, naturally enough, open-ended and fairly subjective (that's why analogies are so popular in *poetry*). Couple this point with my earlier hint that the individuation of philosophical problems is none too clear, and the reader won't be blamed for thinking that the ground rules of this debate between me and McEvoy are too ill-defined to be *resolvable*. Unsurprisingly, perhaps, I'm going to try to show this is false; that the matter isn't just resolvable, but resolvable in my favor.

Let's start with the background motivation for noting a parallel with traditional skepticism: it's a form of *dismissivism*. Correspondingly, the background motivation for denying the parallel to skepticism isn't a misguided hope for some credit for originality; it's the claim that even if skeptical considerations are *neutralized* (one way or another) that the concern with the existence of mathematical abstracta *remains*. Notice that this neutralization point can hold even if there is an *analogy* between the two forms of argument. After all, surely there is an *analogy* (a rather close one) between what we might call *Evil-Demon skepticism* and *Dreaming skepticism*. Both involve possibility-scenarios (I might be dreaming right now; I might be in the grip of evil demon); nevertheless, it's obvious—or should be—that arguments that neutralize the threat of Dreaming skepticism might still leave Evil-Demon skepticism intact.

Let's start with Benacerraf's problem. The worry, as I've noted, is about a particular kind of *object*, an object that is causally inefficacious, causally insensitive, and not in space and time (and maybe that's weird in a whole bunch of other ways as well). It should be clear that even if one has *neutralized* the standard skeptical concerns McEvoy invokes (inferences from phenomenologically-identical experiences that encapsulate all our relevant evidence) that concerns about these metaphysically peculiar objects remain. Imagine a scientist, for example, who postulates a kind of particle that is causally neutral in a similar way. It would be insensitive to the concerns of his opponents, *philosophically insensitive,* to invoke analogies to standard skepticism to undercut his concerns. To use faintly technical terminology: this would be to *trivialize* those concerns. In short, Benacerraf's problem is rooted in the presumed metaphysical peculiarities of abstracta that make them immune to epistemic inroads. To compare this to skeptical scenarizing where one eliminates all our epistemic tools *altogether* (so that every object is now immune to epistemic inroads) misses the philosophical point.

Okay, what about the epistemic role puzzle? I would have hoped that the differences that matter between the concerns it raises and standard skeptical concerns would be even more obvious in its case. After all, the official concern of the puzzle is this: notice that our standard *epistemic practices* have certain accompaniments: methods of recognizing the *epistemic artifacts* that our means of access to the objects in question have *because of* those means of access.

Indeed, the facts about this are intricate and subtle enough to give rise to *sciences of* those means of access. Surely, drawing an analogy between these considerations and external-world skepticism is to miss the philosophical point: In the one case, we are focusing on facts about our epistemic access, facts that count *unfavorably* towards existence claims for mathematical abstracta. In the other case, we are undermining our methods of epistemic access altogether. The result, of course, counts unfavorably towards the existence of *anything* in the external world: this is *analogous*, in the usual sense of the word. I'd like to hope, however, that the *disanalogy* between undermining our methods of epistemic access altogether and noting that those very methods don't accommodate certain purported objects *even if left intact* would be taken note of as well.

8. *Benacerraf's problem and the epistemic role puzzle: Close but different nevertheless.*

As I mentioned, the epistemic role puzzle can be seen as arising from Benacerraf's problem by flipping the focus of concern: not how do we manage to know about those objects, but instead, why isn't epistemic access to these objects a topic of mathematics, or a part of an ancillary science of mathematics? Nevertheless, in practice, bringing the puzzle against an approach in philosophy of mathematics can often be reconceptualized as bringing the problem against that approach. Doing so doesn't show they're the same concern; it only shows they live in the same neighborhood.

McEvoy (2012, 10-11) writes of my discussion of apriorist varieties of Platonism, when I complain that "no explanation is possible at all for how we can have a priori knowledge of ontologically independent objects …"[22]:

[22] From Azzouni 2000, 237.

> In these passages, Azzouni raises the (serious) problem for the platonist's epistemology: how is it that human cognitive processes can reliably yield knowledge of a realm of abstracta which is entirely ontologically independent of those process? If this sounds familiar, it is—because we are now facing … Field's version of the Benacerraf problem. Interpreting ERP as a demand for how our beliefs are sensitive to the facts obtaining in the platonic realm does yield a significant challenge to the Platonist … but it does so at the cost of reducing ERP to the Benacerraf problem.

I take myself to have *already established* in the foregoing pages of this paper that the puzzle and the problem are distinct, both in how they can be responded to, and in the sorts of issues that they make philosophically salient. My only job, therefore, is to delineate how these concerns dovetail in this particular case. The epistemic role puzzle tells us that there is no epistemic role for abstracta, in contrast with (some of) the items posited in the empirical sciences. It immediately follows that we have no *reliabilist story* for how we know what we (purportedly) know about abstracta *because we have no epistemic story at all*.[23] Benacerraf's problem runs the objection more directly: We have no reliabilist story for abstracta like we have in the empirical sciences. That's a *problem*.

9. *Some concluding remarks*.

[23] Furthermore—although I wasn't running this argument in 2000, see footnote 9—if our criterion for existence is mind- and language-independence, we can simply deny, on these grounds, that the purported objects exist.

If you're a genuine (card-carrying) nominalist, Benacerraf's problem isn't going to quite do the job you need done. The contemporary proponents of Field's program show this by blithely countancing abstracta, such as spatial points and regions—which they regard as meeting the challenge Benacerraf's problem successfully poses for more "remote" abstracta; so too, philosophers who think that mathematical statements can "index" nominalistic content—where such content involves spacetime-embodied abstracta—presumably feel that Benacerraf's problem is met by their abstracta, precisely because those items are spacetime-embodied.[24] The epistemic role puzzle is more unforgiving, so I've argued, requiring more of spatial points and regions—that they not only be "in principle" perceivable but that epistemic access to them actually play a role in the establishing of mathematical truths.[25]

Apart from this, however, I've argued that the epistemic role puzzle is a necessary component in at least one argument for nominalism because rejections of correspondence truth and Quine's criterion are insufficient all on their own to refute the platonist.

Bibliography

Azzouni, Jody (1994) *Metaphysical myths, mathematical practice: The ontology and epistemology of the exact sciences*. Cambridge: CUP.

Azzouni, Jody (2004a) The derivation-indicator view of mathematical practice. *Philosophia Mathematica* (3) Vol. 12, 81-105.

Azzouni, Jody (2004b) *Deflating existential consequence: A case for nominalism*. Oxford: OUP.

Azzouni, Jody (2006) *Tracking reason: Proof, consequence, and truth*. Oxford: OUP.

Azzouni, Jody (2007) Ontological commitment in the vernacular. *Noûs* 41: 204-26.

Azzouni, Jody (2010a) *Talking about Nothing: Numbers, Hallucinations, and Fictions*, Oxford: OUP.

Azzouni, Jody (2010b) Ontology and the word 'exist': Uneasy relations. *Philosophia Mathematica* (3) 18, 74-101.

Azzouni, Jody (forthcoming) Deflationist truth. In (Michael Glansberg, ed.) *Handbook on truth*.

[24] See Daly and Simon 2009.

[25] I should note, by-the-by, that I *don't* think that spacetime points or regions are "in principle" perceivable; but I've left that point aside for the sake of discussion.

Oxford: OUP
Balaguer, Mark. (1998) *Platonism and anti-Platonism in mathematics*. Oxford: OUP.
Baker, Alan (2003) Does the existence of mathematical objects make a difference. *Australasian Journal of Philosophy* 81:2, 246-264.
Baker, Alan (2010) A medley of philosophy of mathematics: Review of Philosophy of mathematics: Set theory, measuring theories, and nominalism edited by Gerhard Preyer and Georg Peter, *Metascience* 19(2), 221-224.
Barrett, Harrison H., Kyle J. Myers (2004) *Foundations of image science*. New York: John Wiley & Sons, Inc.
Benacerraf, Paul (1973) Mathematical truth. Reprinted in (Paul Benacerraf, Hilary Putnam, ed.) *Philosophy of mathematics: Selected readings*, second edition. Cambridge: CUP, 1983, 403 -420.
Carey, Susan (2009) *The origin of concepts*. Oxford: OUP.
Daly, Chris, and Langford, Simon (2009) *Mathematical explanation and indispensability arguments*. Philosophical Quarterly 59: 641-658.
Field, Hartry (1980) *Science without numbers*. Princeton, NJ: Princeton University Press.
Field, Hartry (1989) Introduction. In *Realism, mathematics & modality*. Cambridge, MA: Blackwell Publishing, 1-52.
Hacking, Ian (1983) *Representing and intervening*. Cambridge: CUP.
Horwich, Paul (1998) *Truth*, 2nd edition. Oxford: OUP.
Lynch, Michael P. (2004) Truth and multiple realizability. *Australasian Journal of Philosophy* 82(3): 384-408.
Maddy, Penelope (1990) *Realism in mathematics*: Oxford: OUP.
McEvoy, Mark (2012) Platonism and the 'epistemic role puzzle'. *Philosophia Mathematica* (III) 20:3, 289-304.
Raley, Yvonne (2008) Jobless objects: Mathematical posits in crisis. In *Philosophy of mathematics: set theory, measuring theories, and nominalism*, ed. Gerhard Preyer, Georg Peters, 112-131. Frankfurt: Ontos-Verlag.
Rock, Irvin (1997) *Indirect perception*. Cambridge, MA.: The MIT Press.
Strobel, Howard A. and William R. Heineman (1989) *Chemical instrumentation: A systematic approach*, third edition. New York: John Wiley & Sons, Inc.