

Logic and Knowledge

Edited by

Carlo Cellucci, Emily Grosholz
and Emiliano Ippoliti

**CAMBRIDGE
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P U B L I S H I N G

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CHAPTER TWO

NOMINALISTIC CONTENT

JODY AZZOUNI

SUMMARY Certain philosophers have recently responded to the indispensability problem by treating indispensably used statements as proxying for contents without undesirable ontological commitments. Thus, the ontological commitments of the indispensably used statements aren't taken literally. Instead, those ontological commitments are recognized as only those of the contents that these indispensably used statements stand for. This approach is here considered specifically as a nominalist response to the indispensability problem. It's argued that either the proxy content approach doesn't meet certain requirements, or that it collapses into a very different position where the indispensably used statements are literally understood, and Quine's criterion is instead denied.

KEYWORDS nominalistic contents, Quine's criterion, Quine-Putnam indispensability, proxy-contents, one-one demand, nominalism

1. Introduction: The indispensability problem

Let U be a class of ontologically undesirable entities. Different philosophers disagree about what's in U , but some examples are abstracta, fictional entities, and theoretical entities: different philosophers deny the existence of, or deny that we should be committed to, abstracta, anything other than observable entities, literary fictions, etc.

Strictly speaking, the constant terms and quantifiers are seen as problematic—not the entities themselves—because there are no such entities on the (various) views of these philosophers. What there are, however, are undesirable quantifications and constant terms in the statements that we indispensably use—in scientific theories and in ordinary life. For ease of expression I continue to describe the issue in

terms of a class U of undesirable entities that a given philosopher wants to eliminate quantification over.

Quine (1953) was the architect of the overall framework within which a particular philosophical knot has flourished, one I call (following tradition) *the indispensability problem*. In particular, he set in place the *debate markers* that subsequent philosophers have attempted to challenge in various ways. I briefly lay out these debate markers in this section, and indicate some of the ways that subsequent debate has been influenced by Quine's original framework.

First, there is Quine's assumption that a first-order formalism is the standard of what an interpreted language quantifies over. Not all contemporary philosophers agree, but it is a broadly accepted default standard. A burden of proof is therefore undertaken by those deviating from it. Motivating this default standard is the relative transparency of the standard interpretation of first-order idioms: that *quantificational tissue* appears nowhere else in that formalism. In particular, quantificational structure doesn't implicitly occur in the logical roles of the (first-order classical) connectives, nor is it implicit in the roles of predicates or constant terms.

As a result, both those philosophers who read the range of the existential quantifier as inducing the ontic commitments of an interpreted first-order language and their challengers find those purported commitments particularly visible. Such an ontologically-weighty reading of the first-order quantifiers is assumed by most proponents of the to-be-described proxy-content response to the indispensability problem. Call Quine's criterion (for ontological commitment) the demand that the first-order quantifiers in an interpreted formalism be given an ontologically-weighty reading. A neat debate marker has emerged. One can accept or deny Quine's criterion. (For the record, I've long denied Quine's criterion. See Azzouni (1997).)

Quine allows only a Draconian solution to the indispensability problem (with respect to an undesirable class U of entities). One must *replace* the statements that quantify over undesirable entities with other statements that can play the same assertoric roles but that don't so quantify over these entities. Otherwise one is stuck with the statements and therefore stuck with the ontological commitments of those statements by virtue of one's indispensably assertive use of those statements in the sciences (and in ordinary life). An assertive use of a statement is the straightforward utterance of it to represent how things are by means of it. "Assertion" of course includes written formulations of statements, as these occur in the sciences. Quine's only allowed solution to the indispensability problem is

what (following tradition) I call *paraphrase*. (See Azzouni (2009) for details and for refinements.)

This Draconian solution of total replacement is forced in Quine's view because one can't use a statement assertively while denying one's commitments to the logical implications of that statement. One isn't allowed to whisper off-stage, after asserting S: "but of course I don't really mean D," where D is some undesired implication of S.

A second debate marker has emerged. This is to claim, as a number of philosophers have recently, that we *can* assertively use a statement S but nevertheless deny some or all of what it implies. For example (Yablo (2001)), we can treat S as inextricably involving metaphors, as a statement that isn't to be taken literally. Instead, when assertively used, it stands for some other content C that is the literal item that we intend to assert (and commit ourselves to). However, C is a content that can't, for one reason or another, be explicitly used by us instead of S. Perhaps our language is riddled with metaphors, for example, and we can't assertorically use any statement literally.

Or perhaps, although we assert S, we can weasel out of one or more of its implications by explicitly denying them (Melia (2000)). Strictly speaking, then, we have uttered a contradiction. But, we don't ever mean to commit ourselves to a *contradiction*. Rather, the contradiction stands for something else, a consistent content C that we can't (for some reason) utter explicitly.

Or, perhaps we're nominalists, and we refuse to commit ourselves to what we describe as the non-nominalistic content of S, even though there is no way for us to avoid explicitly asserting S. We nevertheless take S to stand for its own non-nominalistic content C, however that's to be characterized (Balaguer (1998), Rosen (1991)). It's the unarticulatable C that we're really asserting and committing ourselves to, even though we can't explicitly express C directly except by our use of S.

Or, perhaps we're constructive empiricists (van Fraassen (1980)), and we claim that although we must assertorically use a theoretically-laden S in our science, we're only committed to its empirical adequacy and nothing more. On one interpretation of what a commitment to empirical adequacy means, we're thus not committed to S. Instead, we're committed to the observational content C of S. It's C that we assert and commit ourselves to by means of our use of S. And this is so despite an inability to articulate C explicitly but only indirectly by means of our use of S.

On every one of these views, although it's conceded that the assertoric use of S can't be eliminated, it's nevertheless assumed that there is something else C—something we can't articulate—that's all and only

what we're committed to when using S. The indispensably used S, that is, *proxies* for something else, something that I call the *intended proxy-content C of the indispensably-asserted statement S*. Despite philosophers using very different language (as just illustrated), all the just-described responses to the indispensability problem can be subsumed under the descriptive label *the proxy-content approach to indispensability*.

Notice that strictly speaking, the various proxy-content proponents are not rejecting Quine's criterion. They may appear to be doing so because they may appear to be allowing themselves the assertive use of certain statements while explicitly denying the existence of what those statements imply exists according to Quine's criterion. However, what's *actually* going on is that the target C of the proxy S is what's strictly being evaluated for ontological commitments. The indispensably asserted statement S itself is treated as only assertively standing in for a target content C, and (presumably) C doesn't so quantify over any entities from the undesirable class of entities U.

Notice that if the proxy-content proponent were to actually reject Quine's criterion, the motivation for introducing proxy-contents as what we are *really* saying (and committing ourselves to) when we otherwise assertively use statements would vanish altogether. This is because one could instead take the indispensably asserted statement S literally while simultaneously denying the ontological commitments apparently due to S's quantifier commitments. One would be licenced in doing this by the rejection of Quine's criterion.

On one interpretation of the debate between the Quine-criterion denial and the proxy-content responses to the indispensability problem on the one hand, and the paraphrase approach on the other, it's assumed by the opponents of paraphrase that paraphrase won't work—at least for the class of entities U that they are individually concerned with. It's assumed, that is, that quantification over the class of entities U is *indispensable*. This is a substantial claim. In particular, ruled out are strategies of eliminating explicit quantification over abstracta by the use of higher-order logics or by the use of other logical formalisms, such as modal ones. These strategies are excluded because there are translation-theorems available that show that the use of such formalisms is equivalent to the use of various first-order languages that quantify over abstracta of one sort or another.

In Azzouni (1994, Part I, § 3), I illustrated such a translation-theorem for classical higher-order logics. It should be noted that such translation-theorems are in general required to preserve the metalogical properties of the formalisms that are so translated into first-order idioms. This is part of

the brief that treating first-order formalisms as the standard for the presence of quantificational structure is cogent and justified. I should also point out that there are variations of first-order languages in which names or predicates play an ontological role that's independent of the quantifiers. The translation-theorems I'm speaking of apply to these variants as well.

Although I don't argue further in this paper for the indispensability assumption with respect to the particular set of undesirable entities U (abstracta) that's the focus of section 2, I should note that some philosophers (e.g., Field (1989), Balaguer (1998)) have suggested that languages that employ more substantial idioms (over and beyond what's available in the first-order setting), such as logically-primitive modal possibility operators, or predicates that are fixed in their interpretation over a range of models, should not on that basis alone be convicted of assuming additional ontological commitments. Such a stance, however, is ill-advised for the denier of any class of entities U . Although languages with primitive logical idioms aren't to be immediately convicted of additional hidden ontological commitments, that's only because what ontological commitments they have are invisible pending an analysis of the logical powers of those idioms. Such an analysis is enabled, as indicated earlier, by translation-theorems: the construction of first-order formalisms with identical metalogical properties, so that the sources of the logical powers of the primitively-adopted idioms can be specified in terms of the fixing of the range of special predicates and terms as well as the presence of various kinds of entities in the domain of the quantifiers. This makes visible the logical powers of otherwise primitive idioms, and in particular exposes their additional ontological commitments, if any.

It should be added that a second interpretation of the debate between the Quine-criterion denial and the proxy-content responses to the indispensability problem on the one hand, and the paraphrase approach on the other is available as well. This is for the opponents of the paraphrase approach not to adopt the more contentious premise that paraphrase doesn't work for one's chosen class of undesirable entities U , but to instead argue on independent grounds that Quine's criterion is false or that proxy-content assertions are in any case appropriate. One can then argue that paraphrase isn't necessary as a tool (to begin with) to avoid ontological commitments, and that that's so even if appropriate paraphrases *are* available. For the record, however, I think that genuine principled nominalistic or constructive empiricist positions (for example) cannot be supported by the paraphrase approach: the needed paraphrases don't exist. Indications of why this is so will be given in subsequent sections of this paper, especially in section 2.

2. Nominalistic content

For illustrative purposes, I now turn to a specification of U that a particular form of nominalism is concerned with. The *serious* nominalist to be considered here denies that anything other than “concreta” exist. Spacetime points are items that some philosophers claim are acceptably construed as concreta (e.g., Field (1980, 3), Field (1989), Melia (1998)). The nominalism currently under exploration, however, and consequently the resulting notion of nominalistic content that will here be available, disallows the inclusion of spacetime points, functions, etc.

I won’t now fully revisit my previous discussions (e.g., Azzouni (2009)) motivating the view that all such entities, especially spacetime points and fine-grained regions of space, shouldn’t be treated as nominalistically-acceptable, but instead should be seen as the terminological result of the application of a mathematical formalism to antecedent concreta. But it’s worth noting that the rejection of spacetime points or sufficiently fine-grained spatial regions isn’t in any case the rejection of one or another version of space-time substantivalism. The point is a different one. It’s that it can’t be presumed that when a physical entity is classified as concreta, as nominalistically acceptable, that therefore the *quantifilia* of its full description in mathematical language—within physical theories—is nominalistically acceptable as well. In particular, spacetime itself may well be nominalistically acceptable, and even regions of such, although certain “parts,” e.g., points, lines, etc.—that are the fellow-travelers of the imposition of a coordinate framework upon it (in order to enable the mathematical deployment of various operators and functions)—*aren’t* nominalistically acceptable.

I should also point out that this exclusion of coordinate-entities from the class of concreta is fully echoed in the ontological attitudes manifested in ordinary scientific practice. The positing of new physical entities, even ones characterized as singularities, is always accompanied by attempting instrumental access to such both to verify their existence and to verify their physical properties. This isn’t true of those entities, or of the properties of any entities, that are introduced by the application of mathematical tools—for example, by the use of coordinates. Rather, the latter sorts of entities and the latter sorts of properties are entirely stipulated by the mathematical formalism that’s been adopted. Spacetime points are in the latter category; gravitons are in the former. This point bears heavily against those philosophers who want to include spacetime points or even spacetime regions among the acceptable entities that a nominalistic language can quantify over.

It should be noted, however, that the status of an item quantified over can change; it can be treated as purely a matter of the application of the mathematics at one time, and then it can come to be treated later as a genuine physical posit by scientists. I should stress again that the status of an entity quantified over by a physical theory and by its proprietary mathematics is indicated not by the nature of its properties but instead by the attitudes of the science towards it—in particular, it turns on whether attempts are made to directly determine the posit's properties and its existence or whether instead merely confirmation of the whole theory suffices.

A second class of entities to be excluded from the range of the quantifiers in nominalistic contents are properties and relations. Some philosophers attempt to redescribe properties and relations as concreta. Indeed, it's always possible to redescribe the interaction of two objects instead as interactions between their properties; it's always possible to redescribe concreta in such a way that properties or higher-order collections of properties are endowed with the qualities of those concreta, e.g., by directly treating concreta as "bundles" of properties. Simultaneously, such "properties" and "relations" may be endowed with other qualities otherwise generally absent from concreta. It's an assumption of this paper that all such metaphysical manipulation is technical trickery, that nominalistically-acceptable concreta are too simple in their metaphysical profiles to allow the inclusion among them of philosophically-redescribed properties and relations—e.g., causally-sensitive properties and relations. For similar reasons, I exclude from the class of concreta those entities from other logically possible worlds or the possible worlds themselves. A bit of terminology that I've already started using is the pair of terms, "concreta" and "abstracta." It has become, hopefully, clear enough for the purposes of this paper what sorts of entities I intend to be sorted into each category.

The particular version of the proxy-content approach with respect to nominalistic contents that will subsequently be the topic of analysis takes this form: Each statement *S* of a scientific language proxies for a nominalistic content *C*, where *C* doesn't quantify over anything other than concreta. Given such a notion of nominalistic contents, we can ask: Are there enough nominalistic contents to go around for all our scientific statements?

Why is this an issue? Because each statement *S* of a scientific language is distinguishable from the other ones by its role—both deductively and representationally. That is to say, each scientific statement *S* is used assertorically to represent states of affairs that are taken to be the case (or

not the case). They are also used in deductions where the conclusions of those deductions must be asserted not conditionally relative to the assumptions used to derive those conclusions but independently of them. This requires the assertoric use of scientific statements in deductions. And finally, their relationships to evidence differ—both the evidence for them, and the evidence that they provide for other statements.

The foregoing points motivate what I call the *one-one demand* on nominalistic contents. At least this much is required of nominalistic contents: that scientific statements with distinguishable assertoric and deductive roles have distinctive nominalistic contents. This, notice, is a uniqueness and existence demand on nominalistic contents. In general, the one-one demand must be met by any proponent of a proxy-content response to the indispensability problem with respect to a class of undesirable entities U . It must be *proven* by a proxy-content proponent that a particular class of nominalistic contents meets the one-one demand.

There is a tight analogy here with solutions to systems of differential equations. Where such solutions can't be exhibited, one is mathematically driven to attempts at characterizations of such solutions without being able to exhibit them explicitly. First required, however, is a proof that such solutions exist and (with respect to a set of parameters) are unique.

It should also be noted that proxy-content proponents rarely (or never) attempt to prove that their particular contents meet the one-one demand. Instead, what's offered are *examples* of statements and their purported target contents, e.g., "The number of Martian moons = 2," "There are exactly two Martian Moons" (Rosen (1991)) or "The average star has 2.4 planets," with a targeted content of an infinite set of statements constructed in a certain way (Melia (2000)). But such examples don't even offer a *recipe* for understanding why all indispensably utilized statements that quantify or refer to undesirable entities have target nominalistic contents that they can be taken to stand for, let alone different ones for proxy statements that differ in their assertional and deductive roles.

One last point to round out this section. Notice that nothing in the one-one demand is specific to nominalistic contents; it's a demand that any proxy-content proponent must show is met with respect to any class U of undesirable entities.

3. An example of a point-particle trajectory model

I turn now to providing a toy example of a set of scientific statements, and exploring attempted characterizations of their nominalistic contents. Consider the following class of models: A number of point-particles

moving about over time, and nothing else. Presume for the sake of argument that certain properties of the point-particles, in particular their relative locations, relative velocities and relative accelerations, can be determined empirically. Presume further that the point-particles are real, and these properties of theirs are physically genuine. (In describing them as “point-particles,” however, licence is being taken to enable certain empirical laws to be statable. In reality, they aren’t really point-particles, or the particular issue of their actual extension has been left aside.) It’s also presumed for the sake of illustration that none of the following are real, that is, that none of the following are to be quantified over in nominalistic contents: spatial regions, temporal regions, distances, spatial points, temporal points. In characterizing nominalistic contents, therefore, we can quantify (and indicate by constants) only the point-particles themselves. Everything else must be captured by coined predicates that hold of specific point-particles or of groups of such.

A scientific language, of course, quantifies over much more. In particular, such a language possesses spatial and temporal variables in addition to the quantifiers that range over the point-particles themselves. This allows us to pose the issue of the relationship between the scientific sentences that in general possess spatial and temporal variables, as well as variables that takes point-particles as values, and nominalistic contents with quantifiers that range *only* over point-particles. Each point-particle trajectory model poses in a fresh way the issue of the relationship between nominalistic contents and the statements they are the nominalistic contents of. This is because what can be represented as occurring in each model can differ greatly.

Let’s consider one example at length. Imagine three point-particles moving relative to one another in a plane. For purposes of imposing a coordinate system to facilitate descriptions of their relative motions, one of the point-particles is treated as fixed in motion and is called \mathbf{O} . The other two are labeled \mathbf{a} and \mathbf{b} . It’s *empirically* discovered that the following equations (given a certain choice of coordinate system) capture the relative motions of the point-particles over time when \mathbf{O} is treated as always fixed at the coordinate $(0, 0)$, the first moment 0 is taken to be that moment when all three particles are located at the coordinate $(0, 0)$, and the motion in the system first begins: $\mathbf{x}_a(t) = t^2$, $\mathbf{y}_a(t) = 0$; $\mathbf{x}_b(t) = 0$, $\mathbf{y}_b(t) = \sin^2 t$. That is to say, if \mathbf{O} is treated as fixed, then the motions of \mathbf{a} and \mathbf{b} are particularly easy to describe: \mathbf{a} is rushing away from \mathbf{O} with ever-increasing velocity, while \mathbf{b} is oscillating with varying acceleration (on a line perpendicular to \mathbf{a} ’s direction of movement) between \mathbf{O} and the distance 1 from \mathbf{O} .

It's an elementary algebraic exercise to relate the motion of **a** at a time to that of **b**, to derive a formula of the distance between **a** and **b** at a time, and it's an elementary calculus exercise to derive formulas for the velocities and the accelerations of these point-particles at a time. All of this information can be expressed, of course, in sentences that quantify over temporal points and spatial points. But what does this information look like if we have only predicates to work with over and above the three point-particles, **a**, **b** and **O**? Consider as an illustration the statement:

$$(1) \quad (t)d(\mathbf{a}, \mathbf{O}, t) = t^2.$$

(1) describes the distance d between **a** and **O** as a particular function of the elapsed time. The nominalistic content of (1) can apparently be captured by an uncountable list of sentences of the following form:

$$(2) \quad d_i(\mathbf{a}, \mathbf{O}) \leftrightarrow T_i(\mathbf{a}, \mathbf{O}).$$

Each predicate $d_i(x, y)$ holds of x and y if and only if the distance between x and y is t^2 ; the predicate $T_i(x, y)$ holds of x and y only when they are at the moment t . Thus, moments are treated as properties that sets of point-particles have. That is, a particle at a moment is treated as that particle having a moment-property, just as a particle being at a position is treated as that particle having a position-property. That a particle lasts so long and no longer is treated as its having a certain (uncountable) collection of moment-properties. That is, we treat a particle at a moment M being at a certain position P as having a particular predicate PM holding of it, where such a predicate holds of the particle only if it is in P at M . Thus trajectories of particles can be captured by uncountable collections of such predicates that hold of them.

The uncountable *family* of statements (2) (for variable t ranging over the positive real numbers ≥ 0) may, it seems, be taken as the nominalistic content of (1). The set of statements (2) are of course inexpressible except through the use of their proxy (1). So this seems to be precisely the kind of example that motivates the proxy-content strategy with respect to nominalism. The nominalistic statements we want to be committed to can't be stated except indirectly, by using a statement that quantifies over temporal and spatial points.

(1) seems to have important content that isn't captured by (2), however, content that we want to express even though we don't want to be committed to the spatial and temporal posits that (1) commits us to. My

suggestion is that (2) falls short in expressing what we need expressed. Here's why.

First, we can't include the indexing of the predicates of (2) as part of the content of that collection of statements. I'm speaking here of the indexing that allows us to assimilate each subscript t of a distance predicate d_t , with the corresponding subscript of the temporal predicate T_t , and to assimilate both of these with the corresponding moment t . These assimilations are entirely illegitimate, if they are to be taken to be parts of the nominalistic contents expressed by the sentences of (2) because these predicates, being first-order, are all primitive. If they aren't treated as primitive, notice, then we have left the realm of first-order logic because we're helping ourselves to anaphoric relations between predicates. That is to say, by treating the indices that appear in (2) (to enable readability) as genuinely present in the nominalistic contents of (2) is to help ourselves to implicit quantificational structure that's illegitimately (from the first-order point of view) tucked into the predicate-notation. The only reason to index the predicates in (2) is so that the reader can recognize what truth conditions the various predicates have, thus to recognize exactly which uncountable sentences are appearing in (2), and thus to recognize why they are being presented as capturing the nominalistic content of (1). But each predicate that appears in (2) must be genuinely primitive if the set of sentences of (2) are to be truly nominalistic.

In particular, the distance $d_{15}(x, y)$ holds of **a** and **O** only when they have a certain relation to one another, only (that is) when they bear a certain distance to one another, one that we can recognize (relative to an implicit coordinate scheme) to be 15 units. There is no numerical relation between that predicate, however, implicitly or explicitly, to the distance predicate $d_{14}(x, y)$ —despite the appearance of such induced by the notation I've adopted to aid the reader in understanding these predicates. From the point of view of nominalistic content, these predicates bear as much of a relationship to one another as the predicates $G(x, y)$ and $W(x, y)$ do.

That this impacts negatively on the idea that the uncountable set of sentences (2) expresses all the significant content of (1) can be recognized by the other way that a misreading of (2) can be induced. This is that part of our understanding of (1), in fact part of what's required for us to understand (1), are the axioms that topologically describe and order the temporal and spatial points relative to one another. And this is because such topological structure and ordering is crucial to our understanding of how the point-particles *move* over time and through space. Notice how important the ordering of the domain of the temporal variable and the spatial ordering of the range of the functions described in (1) are to our

understanding that the motions in question are *continuous* ones. Continuity is an empirical assumption about these motions: it's open (of course) to empirical refutation that such motions are continuous. But this property isn't captured *at all* by the utilization of primitive predicates as they are employed in the sentences of (2).

4. Snapshot and filmic approaches to nominalistic contents

Consider what can be called trajectory *snapshots* and trajectory *films*. Trajectory snapshots are *sets* of 3-place predicates that hold of the three point-particles when they are at particular moments in particular positions. An uncountable number of these can be taken to be the nominalistic characterizations of the relative movements of **a** and **b** with respect to **O** from positions p_{a_l} and p_{b_l} at time t_l to positions $p_{a_{(l+n)}}$ and $p_{b_{(l+n)}}$ at time t_{l+n} . Alternatively, a 3-place predicate $R_{m,t(m+n)}$ can be taken to hold of a set of particles only if they exhibit a certain set of trajectories from time t_m to time t_{m+n} . That is, the predicate $R_{m,t(m+n)}$ is true of three point-particles if and only if they exhibit a certain filmic trajectory over the time in question.

It can be said, perhaps, on behalf of the proponent of nominalistic contents, that all that's required of nominalistic contents is that they appropriately *ground* the scientific truths that we can express. So, for example, we can imagine that the truth of (1) is grounded by an uncountably infinite set of nominalistic contents involving snapshots, or if preferable (to capture the appropriate ordering and topological relations exhibited by the motions of the point-particles over time) by a single filmic nominalistic content like so:

$$(3) \quad R_{0,\infty}(\mathbf{O}, \mathbf{a}, \mathbf{b}).$$

(3) holds of three point particles if and only if they exhibit, starting from the moment 0, and continuing infinitely forward, the trajectories of motions that are in fact exhibited by the three particles **O**, **a**, **b**. We can say that this single nominalistic content grounds *all* the relevant truths expressible about these particles, that is, their relative positions, velocities, accelerations, and of course, all the functional relations among these properties as well.

These suggestions face a fatal dilemma. If we utilize snapshots or more than one film, the resulting nominalistic contents fail to capture the content of the scientific statements describing motions in the same way that our earlier sets of predicates failed to capture that content. Even if just two

infinite films are used, the topological properties that *connect* the motions that are depicted by the two films fail to be captured. If we instead utilize one film (3), we face a particularly vicious version of a failure to meet the one-one demand because all our scientific truths have a single truth as their (joint) target nominalistic content. A focus on grounding scientific truths in nominalistic contents, that is, must recognize the requirement that those contents underwrite the *distinctive* assertional roles of scientific statements. After all, that's the heart of the proxy-content approach: any use of a scientific statement is to *stand stead for* a nominalistic content. The different assertional uses of different scientific statements therefore requires different nominalistic contents. Grounding all scientific statements in one nominalistic content fails to meet this demand.

5. Stipulating nominalistic contents on the basis of the scientific statements themselves

Perhaps the problem is that we're not being sufficiently *imaginative* in how we've allowed ourselves to introduce nominalistic contents. In particular, we're being overly *stingy* about the predicates available. It's easy to introduce the needed predicates (one might think) by introducing for each scientific statement S, the predicate $N_S(x, y, z)$. Further, it can be stipulated that the truth conditions of $N_S(x, y, z)$ are exactly the same as those of S. For example, the scientific statement

$$(1) \quad (t)d(\mathbf{a}, \mathbf{O}, t) = t^2,$$

has as its content

$$(4) \quad N_{(t)d(\mathbf{a}, \mathbf{O}, t) = t^2}(\mathbf{a}, \mathbf{b}, \mathbf{O}).$$

Such an easy solution to the problem of nominalistic content certainly invites suspicion. In particular, those with Puritanical temperaments will suspect that such a solution has to be too good to be true. In the spirit of Puritanism, I raise some worries to what I'll call the *easy road to proxy-contents*.

Here's a first worry. In general, we don't know how to use the nominalistic content $N_S(x, y, z)$ except parasitically, by using S itself. More specifically, we don't even know what the implications of a statement $N_S(\mathbf{O}, \mathbf{a}, \mathbf{b})$ are, what evidence bears on it, even what it *says*, except by virtue of the statement that's proxying for it. We can't know *any* of these things about the nominalistic targets of proxies except by virtue of our

antecedent understanding of the proxies themselves. This, notice, isn't just a problem for the easy road to proxy-contents, it's a problem that can arise for proxy-contents *in general*—when, that is, they aren't understandable independently of their proxies.

The response by the proponent of nominalistic content is this. What's the problem supposed to be? We can know what the predicate $N_S(x, y, z)$ means by virtue of our understanding of S , and we can know by virtue of the form of any content that it doesn't commit us to anything other than **O**, **a**, and **b**. Precisely here I'm allowing the proxy-content proponent an option that I didn't allow that proponent in Azzouni (2009): that the tail can wag the dog. That is, instead of demanding that we have enough of a grip on the target C of the scientific proxy S to enable us to dictate what logical properties S has, we instead allow the logical properties that S *in fact has* by virtue of its own logical form to dictate the logical properties that its targeted content C is to be stipulated as having. I didn't regard this possibility as legitimate in Azzouni (2009) precisely because I was there understanding the scientific statement to be merely *standing in* for what we were really saying by our use of a scientific statement: its targeted content. But many proxy-content proponents don't seem to have this in mind. They seem to instead want the logical properties of the targeted content to be due to the proxy scientific statement, and indeed, they seem to also treat our understanding of those targeted contents as dependent on our antecedent understanding of the proxies. Consequently (for purposes of argument), I'm leaving aside the normative point (so central to the arguments in Azzouni (2009)) that in scientific practice the logical properties of the target-content of the proxy are to trump the apparent logical properties of the scientific proxy. (Notice, however, that this is an option *not* open to Melia (2000) because his proxies are logically inconsistent.)

We might try raising our “this is far too easy” concern in another way: Can't the easy road to nominalistic contents be generalized to apply to *any* arbitrary undesirable set of entities U ? The answer of the proponent of the easy road to proxy contents is: Yes. Unfortunately for the proponent of the easy road, this answer indicates that proxy contents are now functioning as idle wheels. Let me explain.

This paper started out with the assumption that the proxy-content approach differed from the Quine criterion denial approach because the former *didn't* reject Quine's criterion but instead approached the indispensability problem by introducing a standing-stead-for relation of the indispensably used scientific statement to a targeted content. What we've now discovered, however, is that if we take the easy road to proxy

contents the result is that the proxy *S* does *all* the work. That is, all the logical and evidential properties of any targeted content *C* are *induced* in that targeted content by its proxy *S*. The only active role left for the targeted content *C* is to specify the ontology that we undertake by assertorically using *S* despite appearances to the contrary.

But this amounts, *de facto*, to the denial of Quine's criterion because the existence and uniqueness, and indeed all the logical properties of the targeted contents, except for their ontological commitments, are to be read off of *S*. The power of the quantifiers of *S* to determine the ontology *S* is committed to is the only aspect of the logical form of *S* that's ignored on this approach; but that just *is* to deny Quine's criterion.

6. Placing constraints on nominalistic predicates

If the proxy-content response to the indispensability problem is to remain distinct from a denial of Quine's criterion, then there have to be some constraints placed on the predicates admissible in proxy contents. Let's focus again on the specific case of nominalistic contents. The demand is that only *nominalistically-acceptable* predicates $N(x, y, z)$ are to appear within nominalistic contents. But what are those?

A condition (with teeth) was implicitly placed on nominalistically-acceptable predicates in sections 3 and 4 because the primitive predicates there considered were restricted only to ones that describe relative temporal and locational properties, distances, velocities, and accelerations among the three particles, or to snapshots or films of the relative positions or relative movements of all such particles at a time or over time segments. Why just these predicates (and predicates that can be constructed out of such)? The thought goes back to the characterization of nominalistically acceptable *objects* that was given in section 2. Scientific practice, recall, focuses on determining the properties of certain entities, but is content to allow other entities (and their properties) to be stipulated by the employment of certain mathematics, e.g., by the use of coordinate systems. Only the former entities are nominalistically acceptable. This constraint isn't restricted to entities obviously; it also applies to properties. Some properties of entities are ones scientists genuinely attempt to detect, and others are ones that they simply treat as part of the mathematics. For example, the singularity of an electron (in certain classical theories) is a mathematical imposition of that property on the electron. This isn't the case with its charge.

This constraint on nominalistically-acceptable properties rules out the introduction of the expressive resources available on the proxy-induced

approach of section 5. I myself would press the would-be nominalist to follow the dictates of section 2, and to allow only predicates that characterize empirically detectable properties of empirically detectable objects, as well as predicates constructed out of such. Similar constraints are called for with respect to other classes of undesirable entities *U*—if only to avoid the otherwise vacuous role of proxy contents that results if the easy road to proxy contents is taken. Exactly what the constraints are has to depend on the particular philosophical arguments motivating the particular ontological scruples behind a particular philosopher's choice of *U*.

For example, a constructive empiricist should hardly be willing to introduce all manner of predicates for the desired class of observational contents. And this is for exactly the sorts of reasons that have been given for the restrictions on the class of nominalistic contents: What's observational isn't merely entities, but rather entities exhibiting certain properties or other. Predicates corresponding to unobservable properties are to be allowed within proxy contents on the constructivist view no more than quantification over unobservable entities is. Indeed, in general, the constructive empiricist is committed to a far narrower class of contents than the austere nominalist that's been the focus of this paper. That's because the nominalist allows that concreta (and some of their properties) can be detected instrumentally in ways that go beyond observation. Thus those concreta and some of their properties may be characterized in nominalistically-acceptable contents, although not in the proxy contents that the constructive empiricist allows.

In any case, introducing constraints on the predicates that can appear within proxy contents faces a fatal dilemma—one we've already seen illustrated in the case of nominalistic contents. Either the constraint is empty because it admits as many predicates as the easy road to proxy-contents does (in which case the approach collapses into a denial of Quine's criterion) or the constraint has real teeth in which case it violates the one-one demand on proxy contents.

7. The veil of language objection

The problem with indispensable language is that it obscures our view of the *real* features of the world. It does this in two ways, by introducing quantification over entities that aren't real, and by introducing properties (corresponding to predicates) that aren't had by anything real. These properties can be ones that are attributed to entities that aren't real, but they can also be properties that are attributed to entities that we otherwise

do regard as real and as having (other) real properties. In these ways a veil of indispensable language threatens our access to the real entities in the world, and to what those entities are like.

One appealing aspect of the proxy-content approach to indispensability is that—at least in principle—it seems to offer a way to see the world correctly despite the continued employment of indispensable quantifications over the unreal, and the attributions of properties to real and unreal entities imposed by indispensably asserted statements in ordinary life and in the sciences. On the nominalist proxy-content view, for example, the way the world *really is* is described by the nominalistic contents of statements. Anything more that's involved in the contents of the scientific proxy statements is a projection of language onto the world. If the foregoing considerations presented in this paper are right (including the claim that the paraphrase strategy fails), only the Quine-criterion denial approach to indispensability is still a live option. The question now is how that approach handles the *veil of language objection*. That is, unlike with nominalistic content, although we can recognize that we aren't committed to certain entities, we aren't otherwise able to see which content in our scientific statements tells us about the world and which doesn't. The worry can be put this way. Although we can tell, on the Quine-criterion denial approach to indispensability, which entities are real and which aren't, we don't have a way of distinguishing what they're really doing from what they're not really doing.

The worry is groundless because the Quine-criterion denier has exactly the same access to nominalistic contents that the nominalistic proxy-content proponent has. And *that* means the Quine-criterion denier can evaluate the relationships between scientific statements and their nominalistic contents just as the proxy-content proponent can. The difference in the two views comes down to this. The Quine-criterion denier isn't burdened by the one-one demand because the Quine-criterion denier rejects the proxy approach to indispensable assertion. In particular, no claim is made that in asserting any scientific statement what's actually asserted is its nominalistic content. Rather, the Quine-criterion denier can instead focus on whether (and to what degree) the truth of a scientific statement tracks what's actually happening in the world. Some scientific statements do this more than other statements. But they are all equally asserted by scientific practitioners, and in particular they are all taken by scientific practitioners to be true.

What *is* the case is that the Quine-criterion denier has to admit that truths have degrees of being grounded in the machinations of the real world. Some statements are true precisely because of what's happening in

the world. More precisely, some statements are true because the objects they are about have the properties that those statements attribute to them. That is, the correspondence picture is right about some scientific statements. But other scientific statements are true only in part because of what those statements are about, and what those statements attribute to the things they are about. And some statements may be true despite their being about nothing at all.

In saying all this, the Quine-criterion denier is *not* adopting a pluralistic notion of truth—at least, the Quine-criterion denier doesn't have to do this (and I think the Quine-criterion denier would be wise *not* to do this). The notion of truth utilized can just be a standard Tarskian-style notion that lacks any metaphysical component whatsoever. These are all matters that I've discussed at length elsewhere (Azzouni (2006), Azzouni (forthcoming)). The only point being made now is that the Quine-criterion denier has a lot of resources for circumventing the veil of language concern.

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